Four-Dimensional Guidance Problem with Control Delays

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This paper, assuming steady wind and zero sideslip, presents a discrete-time mathematical model to obtain a control law and three-dimensional flight path to guide an aircraft in a given time from a given initial state (position, velocity and heading) to a prescribed final state subject to the constraints on airspeed, acceleration, and pitch and bank angles of the aircraft. For ease in implementing the control law, the control inputs are assumed to be delayed and are applied in a sequential fashion. The guidance problem is formulated as a discrete nonlinear optimal control problem with time delays in dynamics and a cost functional of Bolza form. With a quadratic penalty function to handle terminal constraints on velocity and heading, a solution technique to the control problem based on conjugate gradient algorithm is investigated. Numerical examples are presented to illustrate the applicability of this approach to solution of a terminal area guidance problem in an automated air traffic control environment.

Nomenclature

x,y,z = aircraft coordinates with reference to a fixed, ground-based coordinate system (Fig. 1)
v = instantaneous speed of aircraft
ψ = heading of the aircraft (measured positive clockwise from north)
θ = pitch angle (measured positive upward from the horizontal)
ϕ = bank angle (measured positive clockwise from the vertical-right turn positive)
a = acceleration of the aircraft
w = wind speed
σ = direction of wind
g = acceleration due to gravity

I. Introduction

To alleviate the present-day air traffic crisis and increase the capacity of the Air Traffic Control (ATC) system, the general recommendation[1] of the ATC Advisory Committee of the Department of Transportation is a high level of automation in the ATC system since the present-day system is essentially a manual one. As a result, the Federal Aviation Administration is developing and incorporating new traffic control equipment into the ATC such as the new Microwave Landing System (MLS) and the recently introduced Automated Radar Terminal System (ARTS III). The MLS, in conjunction with the new area navigation (RNAV) concepts, provides better accuracy in standard approach procedures and greater freedom of trajectory selection for terminal operations. The ARTS III, basically a computer-assisted video system, provides a dynamic on-line alphanumeric display presenting information to controllers directly on radar scope which would previously have been acquired through voice communication. The controllers and pilots, however, must still mace the basic decisions regarding flight control and navigation. Automation of these and other functions, now performed manually, is quite essential to achieve a safe and efficient ATC system.

The purpose of this paper is to present a mathematical model and to investigate a computer-oriented algorithm to synthesize a three-dimensional flight th to guide an aircraft in a given time, T, from a given initial position, altitude, velocity, and heading to a prescribed final position, altitude, velocity, and heading subject to the constraints on velocity, acceleration, and pitch and bank angles of the aircraft. Such a precise time-position control problem, referred to as a four-dimensional (4-D) guidance problem in the literature, has been proposed as an advanced ATC technique. When applied to terminal guidance problems, the final position would be the Instrument Landing System (ILS) gate coordinates, final heading would be parallel to the runway, and terminal time, T, would be the assigned landing time slot.

The problem of flight-path optimization and control of single aircraft has received some attention in the recent literature. MacKinnon[2] presents a quasi-time optimal solution for the ILS acquisition problem. Kishi and Pfeffer[3] develop a technique for guiding an aircraft from an arbitrary point to a given circular flight-path. Hoffman et al.[4] describe a feedback guidance scheme for VTOL aircrafts. Erzberger and Lee,[5] and Pecsvaradi[6] consider the problem of minimum time horizontal guidance of an aircraft in the terminal area under the assumption of constant velocity and altitude. This requires three state variables and one control variable in its formulation. Lee, et al.,[7] and Erzberger and Pecsvaradi[8] present a set of algorithms for automatically synthesizing flight profiles, i.e., the horizontal, vertical, and speed profiles, as well as a time sequence of commands for the 4-D guidance problem stated earlier. The author in Ref. 9 discusses the same problem, posing it as a discrete-time optimal control problem with five state variables and three control variables.

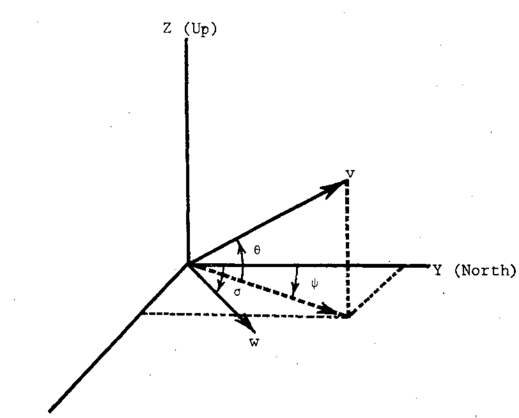

Fig. 1 Aircraft and wind velocities in Cartesian coordinates.

Received September 19, 1974; revision received November 13, 1975. This work was supported by NASA under Grant NGR-21-119-002. The author wishes to thank A. W. Raine and J. H. Stripling of NASA Wallops Flight Center, for many discussions and for providing computer facilities. Many helpful comments of the reviewers are gratefully acknowledged.

Index categories: Aircraft Navigation, Communication, and Traffic Control; Navigation, Control, and Guidance Theory.

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This paper investigates the 4-D guidance problem with a discrete-time model, but with time delays incorporated in the control variables whereby only one control input is to be changed at a time as against simultaneous manipulation of all control inputs in a conventional discrete-time model of multivariable control problems. It is believed that the sequential nature of control components has practical advantages in that it makes manual implementation of control law easier and tends to reduce the time lag in the implementation process. The author is not aware of this type of approach to a multivariable control problem in the literature.

II. Mathematical Model

A. System Dynamics

Assuming coordinated turns and zero sideslip, the dynamics of an aircraft may be represented by the following state equations[10]

$$\dot{x} = v \cos\theta \sin\psi + w \sin\sigma \tag{1}$$

$$\dot{y} = v \cos\theta \cos\psi + w \cos\sigma \tag{2}$$

$$\dot{z} = v \sin\theta \tag{3}$$

$$\dot{v} = a \tag{4}$$

$$\dot{\psi} = (g/v) \tan\phi \tag{5}$$

In order to simplify the model, it is assumed that w and σ are constants throughout the altitude and time range under consideration, and the pitch angle θ is small enough so that $\cos\theta \simeq 1$. The velocity of wind at mean altitude may be used as a good approximation here. With a view to obtain a discrete-time sequence of command inputs and waypoints along the terminal approach route, the total time interval T is broken into a number of discrete intervals, say, N. If τ is the duration of each interval, then $N\tau = T$. Further, it is assumed that only three control variables, namely bank angle ϕ, acceleration (throttle setting) a, and pitch angle θ are at the pilot's disposal and these variables are piecewise constant. That is, for all n and time t such that $n\tau < t < (n+1)\tau$ and $0 \le n \le N$, $\dot{\phi} = 0$, $\dot{a} = 0$, and $\dot{\theta} = 0$.

In view of the foregoing assumptions, one can obtain the following derivatives for all n and t such that $n\tau < t < (n+1)\tau$ and $0 \le n \le N$ from Eqs. (1-5)

$$\ddot{x} = a \sin\psi + g \tan\phi \cos\psi \tag{6}$$

$$\ddot{y} = a \cos\psi - g \tan\phi \sin\psi \tag{7}$$

$$\ddot{z} = a\theta \tag{8}$$

$$\ddot{v} = 0 \tag{9}$$

$$\ddot{\psi} = -\dot{\psi}\, a/v \tag{10}$$

$$x^{(3)} = \dot{\psi}\, \ddot{y} \tag{11}$$

$$y^{(3)} = -\dot{\psi}\, \ddot{x} \tag{12}$$

$$z^{(3)} = 0 \tag{13}$$

$$x^{(4)} = -\dot{\psi}^2\, \ddot{x} + \ddot{\psi}\, \ddot{y} \tag{14}$$

$$y^{(4)} = -\dot{\psi}^2\, \ddot{y} - \ddot{\psi}\, \ddot{x} \tag{15}$$

In an effort to obtain a meaningful and realistic model not too demanding computationally, it is validly assumed that derivatives of order five and above of x and y are negligible since one can easily see that the kth derivative of x (or y) is proportional to the $(2-k)$th power of v, the airspeed.

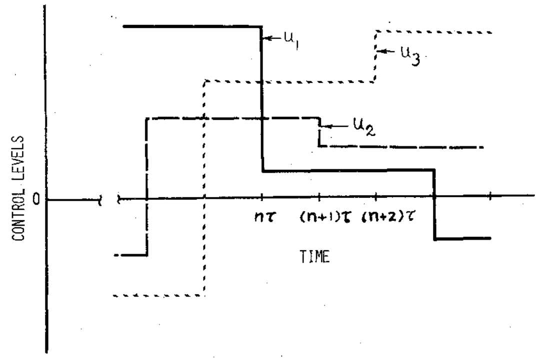

Fig. 2 Nature of control law.

For notational convenience, let us define

$$[x_1^n\ x_2^n\ x_3^n\ x_4^n\ x_5^n]^T \triangleq [x(n)\ \ y(n)\ \ z(n)\ \ v(n)\ \ \psi(n)]^T$$

$$[u_1^n\ u_2^n\ u_3^n]^T \triangleq [\phi(n)\ \ a(n)\ \ \theta(n)]^T$$

where $x(n)$ denotes x at time $t = n\tau$ $(0 \le n \le N)$. X is the state vector (5×1) and U is the control vector (3×1). Using Taylor's Series expansion and including terms up to the fourth-order, one can obtain the following state transition equations

$$x_i^{n+1} = x_i^n + \sum_{k=1}^{4} x_i^{(k)}\, \tau^k / k! \quad \text{for } i = 1,2 \tag{16}$$

$$x_3^{n+1} = x_3^n + x_4^n\, u_3^n \tau + u_2^n\, u_3^n \tau^2 / 2 \tag{17}$$

$$x_4^{n+1} = x_4^n + u_2^n \tau \tag{18}$$

where $x_i^{(k)}$ denotes the kth derivative of x_i^n with respect to time as given by Eqs. (1-15). Equation (5) together with (9) yields upon integration

$$x_5^{n+1} = \begin{cases} x_5^n + g \tan u_1^n\ \ln(1 + u_2^n \tau / x_4^n) / u_2^n & \text{if } u_2^n \neq 0 \\ x_5^n + (g\, \tau / x_4^n) \tan u_1^n & \text{if } u_2^n = 0 \end{cases} \tag{19}$$

For ease in implementing the control law, and to provide for time delays in the pilot's response, communication, and implementation, etc., it is assumed that the control inputs are manipulated one at a time in sequential fashion as opposed to simultaneous manipulation of all control inputs required in a conventional discrete-time model. That is, only one control input is changed at the beginning of each time interval $n\tau < t < (n+1)\tau$ for all n $(0 \le n \le N-1)$, and others are maintained at their previous values. Since there are three control variables, each control input is maintained at one value for 3τ sec. Suppose the first control variable is changed at $t = n\tau$, the second control variable is then changed τ sec later, the third one 2τ sec later, and again the first one is changed at $t = (n+3)\tau$ and so on as shown in Fig. 2. Although any one of the control variables can be used to start the cycle, in this work it is assumed that u_1 (bank angle) is applied first in view of its dominance in the dynamical equations. Note that the values of u_2^0, u_3^0, and u_3^1 are assumed to be specified and can conveniently be set to zero.

B. Formulation of Control Problem

In view of the statements made in the preceding paragraph, the system dynamics given by Eqs. (16-19) may be represented by vector equations

$$X^{n+1} = F^n (X^n, U^n, U^{n-1}, U^{n-2}) \tag{20}$$

and

$$X^0 = A \tag{21}$$

where A is the known initial state vector at time $t=0$. The terminal constraints are

$$X^N = B \tag{22}$$

where b_i's are the desired (specified) terminal states at time T.

Due to the limitations on the capabilities of the aircraft, and to ensure passenger comfort, the following inequality constraints are imposed on the control variables and state variable x_4 (velocity):

$$\left.\begin{aligned} |u_1^n| &\leq \phi_{\max} \\ |u_2^n| &\leq a_{\max} \\ |u_3^n| &\leq \theta_{\max} \end{aligned}\right\} n=0,1,\ldots,N-1 \tag{23}$$

and

$$v_{\min} \leq x_4^n \leq v_{\max} \qquad n=1,2,\ldots,N \tag{24}$$

However, in view of Eq. (18), inequality (24) can be written as

$$v_{\min} \leq x_4^0 + \sum_{i=0}^{n-1} u_2^i \tau \leq v_{\max} \qquad n=1,2,\ldots,N$$

Upon rearrangement, one obtains

$$(v_{\min} - x_4^0)/\tau \leq \sum_{i=0}^{n-1} u_2^i \leq (v_{\max} - x_4^0)/\tau \tag{25}$$

For a given set of maximum, minimum, and initial airspeeds and time interval τ, the upper and lower bounds in (25) are fixed constants; therefore, the inequality constraints (24) on state variable x_4 can be treated as additional constraints on the sums of control variable u_2. In summary, one has the following control constraints:

$$-U_{\max} \leq U^n \leq U_{\max} \quad 0 \leq n \leq N-1$$

$$c_1 \leq \sum_{i=0}^{n-1} u_2^i \leq c_2 \qquad 1 \leq n \leq N \tag{26}$$

The problem is to find control histories

$$u_1^n \text{ for } n=0,3,6,\ldots,N-3 \tag{27a}$$

$$u_2^n \text{ for } n=1,4,7,\ldots,N-2 \tag{27b}$$

$$u_3^n \text{ for } n=2,5,8,\ldots,N-1 \tag{27c}$$

that will transfer the state vector X from A to B in time T, satisfying (20) and (26).

The performance criterion for the control problem is based on the following rationale: as it may not be feasible to obtain control histories (27) that will satisfy all five terminal conditions (22) precisely in time T, one might desire to keep the terminal miss distance, D, given by

$$D = \left[\sum_{j=1}^{3} (x_j^N - b_j)^2 \right]^{1/2} \tag{28}$$

as small as possible while at the same time satisfying the terminal conditions on velocity and heading to within specified tolerances. In essence, one would like to treat the ILS gate (or target) as a sphere of radius R whose center is at (b_1, b_2, b_3). If the optimum value of D is larger than R, it would mean that the aircraft cannot be delivered at the ILS gate (or target) in time T and perhaps a later arrival time should be specified. Furthermore, since $U^n \equiv 0$ for all n implies a steady level flight, one might want to effect the state transfer from A to B in time T with the least possible amount of control efforts. Thus, one could define a cost functional of Bolza form. That is, minimize

$$J' = D^2/2 + k/6 \sum_{i=0}^{N-1} \sum_{j=1}^{3} (u_j^i / u_{j\,\max})^2 \tag{29}$$

with respect to (27) subject to system equations (20-21), control constraints (26), and terminal constraints

$$x_j^N = b_j \qquad j=4,5 \tag{30}$$

where k is a suitable weighting factor and D is as given by (28).

Using a quadratic penalty function to enforce the terminal constraints (30) on velocity and heading, the cost functional J' is modified to J, given by

$$J = J' + \tfrac{1}{2} \sum_{j=4}^{5} p_j (x_j^N - b_j)^2 \tag{31}$$

where p_j's are the positive penalty coefficients that are increased successively until the terminal constraints (30) are satisfied to within specified tolerance levels. In summary, one can state the guidance problem as a discrete nonlinear control problem with time delays in dynamics minimizing (31) with respect to (27) and subject to (20), (21), and (26).

III. Necessary Conditions from the Minimum Principle

As is usually the case with practical control problems, nothing can be said a priori about the existence of an optimal solution to the nonlinear control problem stated in the previous section, and even if a solution could be found, nothing could be said regarding its uniqueness. However, if a solution exists, it must satisfy the necessary conditions specified by the Minimum Principle. Chyung[11] has investigated the optimal control of linear discrete systems containing pure delays in dynamics. Mariani and Nicoletti,[12] using a nonlinear programming approach, have derived necessary conditions for optimal control of discrete nonlinear systems with time delays in dynamics, performance, and constraints.

Define a costate vector Λ^i (5×1) and a Hamiltonian H^i such that

$$H^i = k/6 \sum_{j=1}^{3} (u_j^i / u_{j\,\max})^2 + (\Lambda^{i+1})^T F^i (X^i, U^i, U^{i-1}, U^{i-2})$$

Application of the necessary conditions for optimality yields the following two-point boundary-value problem (TPBVP) which must be solved to obtain a control sequence (27) producing a stationary value of the performance index J

$$\left.\begin{aligned} X^{i+1} &= F^i (X^i, U^i, U^{i-1}, U^{i-2}) \\ \Lambda^i &= \frac{\partial H^i}{\partial X^i} \end{aligned}\right\} 0 \leq i \leq N-1$$

$$X^0 = A$$

$$\lambda_j^N = x_j^N - b_j \qquad j=1,2,3$$

$$\lambda_j^N = p_j (x_j^N - b_j) \qquad j=4,5$$

where the optimal control vectors U^i's must satisfy the gradient relationship

$$G^i = \sum_{j=0}^{2} \frac{\partial H^{i+j}}{\partial U^i} \qquad 0 \leq i \leq N-1 \tag{32a}$$

$$= 0 \qquad H^i \equiv 0 \text{ for } i > N-1 \tag{32b}$$

and the control constraints (26). However, in view of the assumptions made on the control variables in Sec. IIa, the gradient relationship (32) can be written as

$$\sum_{j=0}^{2} \frac{\partial H^{i+j}}{\partial u_1^i} = 0 \qquad i=0,3,6,\ldots,N-3$$

$$\sum_{j=0}^{2} \frac{\partial H^{i+j}}{\partial u_2^i} = 0 \qquad i=1,4,7,\ldots,N-2$$

$$\sum_{j=0}^{2} \frac{\partial H^{i+j}}{\partial u_3^i} = 0 \qquad i=2,5,8,\ldots,N-1$$

IV. Numerical Solution Technique

A numerical solution to the above TPBVP is investigated using Conjugate Gradient (CG) method,[13] chosen for its computational simplicity. Modifications suggested by Pagurek and Woodside[14] are incorporated in the CG algorithm to allow for bounded controls. The problem is solved sequentially with increasing values for the penalty coefficients, p_j's, until the terminal errors on velocity and heading are within specified tolerance levels. At each iteration in the CG algorithm, the step sizes are controlled such that the constraints given by (26) are not violated and a one-dimensional search for the best step size is carried out using a simple quadratic interpolation procedure. Best performance of the CG algorithm is observed when it is restarted with a steepest-descent step after every third (dimension of terminal constraint +1) iteration.

Of the various nominal controls used to start the CG algorithm, the following set of nominal controls generated automatically from the values of initial and desired final states is found to yield good convergence:

$$\left.\begin{aligned} u_1^i &= \arctan\left((x_5^0-b_5)\bar{v}/Tg\right) \\ u_2^i &= (x_4^0-b_4)/(T-\tau) \\ u_3^i &= (x_3^0-b_3)/(T-2\tau)\bar{v} \end{aligned}\right\} \quad i=0,1,\ldots,N-1$$

where $\bar{v}=(x_4^0+b_4)/2$. However, whenever initial and final headings are the same, a nonzero value is chosen for u_1^i's.

Choice of τ, the duration of each discrete interval, can be based on the following observations: for a given value of T, the terminal time, too small a value for τ would mean too much maneuvering and workload on the part of the pilot and a large computation time per iteration, while too large a value for τ, would mean a lesser workload for the pilot but might result in a larger discretization error, and the algorithm might yield unreasonable trajectories or no convergence at all. Further, for $N=T/\tau$, the number of discrete intervals must be a multiple of three. It is believed that a value of 20 to 30 sec for τ can be considered to be a good choice.

The CG algorithm was implemented in the form of a digital computer program whose inputs are the initial and desired final states of the aircraft, desired terminal time, number of waypoints (discrete intervals) on the trajectory, limitations on the capabilities of the aircraft, allowable tolerances on terminal constraints on velocity and heading, and wind speed and direction. The final output is the discrete-time sequence of control inputs and state histories.

The algorithm was tested for various initial and terminal conditions and was found to yield good convergence and reasonable trajectories. The computation time required was, on an average, 25% more as compared with the approach taken in Ref. 9. Alternate solution techniques for the TPBVP and computational hardware might reduce the time required. This increase in computation time does not, however, make the method unsuitable for on-line operation. The following section presents two examples in detail.

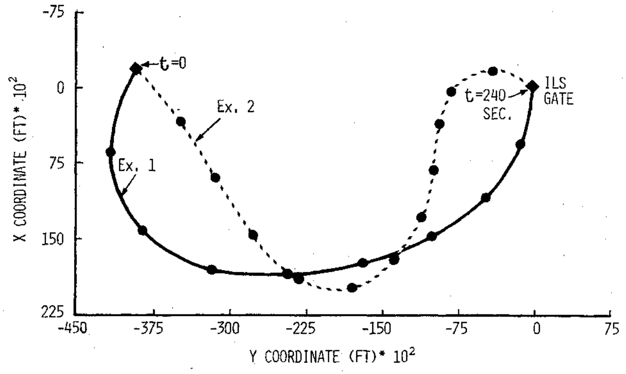

Fig. 3 Horizontal profiles.

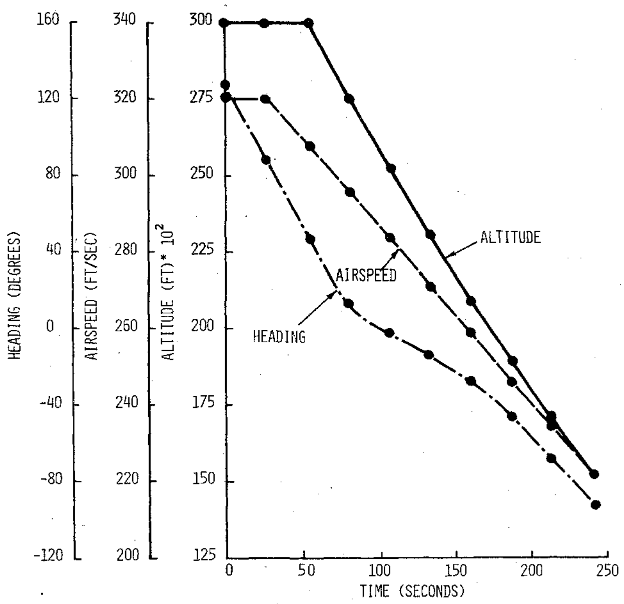

Fig. 4 Altitude, velocity and heading profiles – example 1.

V. Numerical Examples

Example 1

The first example considered here is the one treated in Refs. 7 and 9. Consider an aircraft whose current position is 8 miles from the ILS gate at an azimuth of 202°, a heading of 126°, an altitude of 3000 ft, and a speed of 320 fps. The flight profile to be synthesized is to guide the aircraft to arrive at the ILS gate 4 min later aligned with the runway with a final heading of −90°, and at a final altitude of 1500 ft and speed of 220 fps. The acceleration and deceleration capability of the aircraft is 1 fps^2 and the maximum and minimum speeds are 500 and 220 fps, respectively. Maximum bank and pitch angles are, respectively, 30° and 3°. The wind velocity is zero.

The arrival time (240 sec) is broken into nine intervals of 26.7 sec each. Tolerances of 2% and 2.5°, respectively, are chosen for allowable terminal errors on velocity and heading. The weighting factor, k, for control efforts is chosen to be 10^4. Initial values of 1 and 100, respectively, are chosen for the penalty coefficients p_4 and p_5 and these are increased sequentially by a factor of 1000.

Initial guess on control histories resulted in a terminal miss distance of 22800 ft for this example and was reduced to 83 ft in 12 iterations (computation time 6.1 sec on the Honeywell 625 machine). Computations were terminated as soon as the terminal miss distance fell below an acceptable value. The 4-D command sequence for this problem is summarized in Table 1. The state histories, namely, horizontal, vertical, speed and heading profiles are shown in Figs. 3-4. For this example, the horizontal profile obtained in Ref. 7 consists of segments of minimum radius (maximum bank) turns and straight lines.

Table 1 4-D command sequence for example 1

	Controls			States				
WPN[a] n	ϕ (deg)	a (fps^2)	θ (deg)	x (ft)	y (ft)	z (ft)	v (fps)	ψ (deg)
0	−13.8	0	0	−1574	−39072	3000	320	126.0
1		−.454		6421	−41529	3000	320	88.2
2			−1.71	14085	−38612	3000	308	49.7
3	−3.85			18005	−31788	2760	296	9.6
4		−.462		18540	−24094	2529	284	−1.8
5			−1.70	17552	−16774	2308	271	−13.7
6	−5.93			15153	−10137	2099	259	−26.2
7		−.467		11196	−4720	1899	247	−46.4
8			−1.71	5858	−1233	1709	234	−67.6
9[b]				−65	−44	1528	222	−90.0

[a]Waypoint number. The time of arrival at these points $= n\tau$ where $\tau = 26.7$ sec. [b]Corresponds to ILS gate whose coordinates are $x = 0$, $y = 0$ and $z = 1500$ (desired state).

Table 2 4-D command sequence for example 2

	Controls			States					
WPN[a] n	ϕ (deg)	a (fps^2)	θ (deg)	x (ft)	y (ft)	z (ft)	v (fps)	v_g (fps)	ψ (deg)
0	2.55	0	0	−1574	−39072	3000	320	327	45.0
1		−.490		3588	−34755	3000	320	329	50.1
2			−1.75	9041	−30940	3000	310	321	55.4
3	−16.9			14659	−27708	2814	300	313	60.7
4		−.463		18978	−23403	2633	291	289	22.9
5			−1.74	19766	−17814	2459	281	266	−16.2
6	−5.07			17013	−13447	2291	272	250	−56.6
7		−.445		12713	−10990	2128	263	241	−68.8
8			−1.66	8171	−9660	1971	254	233	−81.5
9	16.3			3636	−9483	1826	245	226	−94.6
10		−.380		−382	−8071	1687	236	215	−49.8
11			−1.56	−1976	−4050	1552	229	218	−34.7
12[b]				−28	28	1429	221	228	44.5

[a]Denotes waypoint number. The time of arrival at these points $= n\tau$ where $\tau = 20$ sec. [b]Corresponds to ILS gate whose coordinates are $x = 0$, $y = 0$ and $z = 1500$ (desired state).

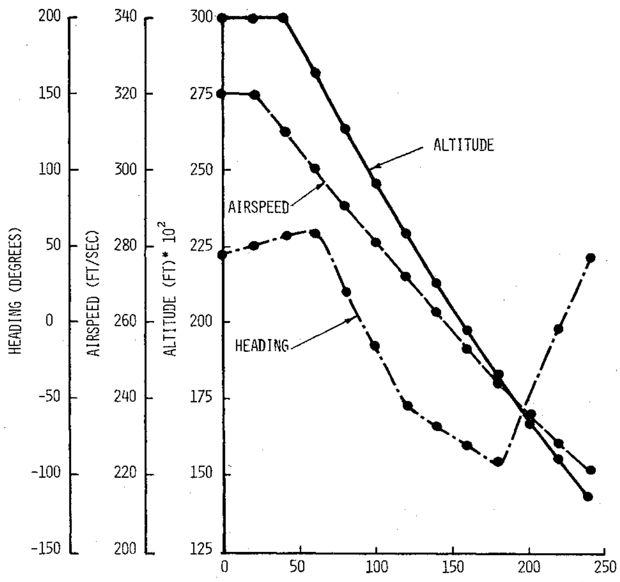

Fig. 5 Altitude, velocity and heading profiles – example 2.

Note that the control inputs are piecewise constant in time and the command table gives the values of the controls only at time instants where they change to new constant values. Positive ϕ means a right turn and positive θ means an ascent (nose up).

Example 2

This example tests the behavior of the algorithm in the presence of wind and under stringent conditions. A wind speed of 22 fps at 90° is imposed and the initial and final headings are changed to 45°, other conditions being the same as in Example 1. For N, the number of discrete intervals equal to nine, the trajectory obtained was rather unsatisfactory requiring two nearly consecutive turns in opposite directions. However, for N equal to 12, the algorithm yielded a reasonable trajectory with slight increase in computation time. The initial guess on controls resulted in a terminal miss distance of 29,500 ft and was reduced to 81 ft in 15 iterations (computer time nine sec). The 4-D command sequence for this problem is summarized in Table 2 where v_g denotes ground speed as given by the relation (see Fig. 1)

$$v_g = [v^2 + w^2 + 2vw \cos\theta(\sigma - \psi)]^{1/2}$$

However, if one assumes $\cos\theta \simeq 1$ for small values of θ and $(w/v)2 << 1$, the above relationship can be approximated by

$$v_g \simeq v + w \cos(\sigma - \psi)$$

The state histories are shown in Figs. 3 and 5.

IV. Conclusion

The 4-D guidance problem that is presently solved manually by pilots and air traffic controllers has been formulated as a discrete optimal control problem in which time delays are introduced in the control variables to obtain a control law that is easy to implement in practice. Penalty functions are employed to handle terminal constraints on velocity and heading. A solution technique based on Conjugate Gradient algorithm has been investigated and implemented in the form of a computer program which computes the guidance law and state histories.

The solution approach to terminal guidance problem discussed herein, requires no precomputation of trajectories and could be implemented on an airborne digital computer or a ground-based computer with data link for on-line operation in an automated ATC environment since the computational and storage requirements are quite modest. Aircraft performance limitations are treated as parameters and hence, this approach could be used for any aircraft.

It should be pointed out that this model neglects aircraft response time delays and control errors, as is the case with other models in the literature on aircraft terminal guidance. The question of how sensitive this method is (compared to others) to these factors needs to be investigated and can really be answered only by flight testing in real-life environment since the dynamical equations (of all models) themselves necessarily involve simplifications of some form. It is believed, however, the sequential nature of control components of this method tends to reduce the time lag in the control implementation process. If, on the other hand, an autopilot were the controller of the aircraft, there appears to be no clear advantage of sequential control actions. Though the model considers acceleration and pitch to be control variables, airspeed and descent/climb rate information can be made available from state histories for pilot manipulation.

References

[1] "Report of the Department of Transportation Air Traffic Control Advisory Committee," Vols. 1 and 2, Dept. of Transportation, Wash., D.C., Dec. 1969.

[2] MacKinnon, D., "Quasi-Optimization of a Localizer Acquisition System," *Proceedings of the 1969 Joint Automatic Control Conference*, 1969, pp. 822-830.

[3] Kishi, F. H. and Pfeffer, I., "Approach Guidance to Circular Flight Paths," *Journal of Aircraft*, Vol. 8, Feb. 1971, pp. 89-95.

[4] Hoffman, W. C., Zvara, J., Bryson, A. E., Jr., and Ham, N. D., "Automatic Guidance Concept for VTOL Aircraft," *Journal of Aircraft*, Vol. 8, Aug. 1971, pp. 637-642.

[5] Erzberger, H. and Lee, H. Q., "Optimum Horizontal Guidance Techniques for Aircraft," *Journal of Aircraft*, Vol. 8, Feb. 1971, pp. 95-101.

[6] Pecsvaradi, T., "Optimal Horizontal Guidance Law for Aircraft in the Terminal Area," *IEEE Transactions on Automatic Control*, Vol. AC-17, Dec. 1972, pp. 763-772.

[7] Lee, H. Q., McLean, J. D., and Erzberger, H., "Guidance and Control Techniques for Automated Air Traffic Control," *Journal of Aircraft*, Vol. 9, July 1972, pp. 490-496.

[8] Erzberger, H. and Pecsvaradi, T., "4-D Guidance System Design with Application to STOL Air Traffic Control," Proceedings of the 1972 Joint Automatic Control Conference, Paper 14-1, pp. 445-454.

[9] Nagarajan, N., "Discrete Optimal Control Approach to a Four-Dimensional Guidance Problem Near Terminal Areas," *International Journal of Control*, Vol. 20, Aug. 1974, pp. 277-288.

[10] "Definition of a Terminal Area Air Traffic Model for Studies of Advanced Instrumentation and Control Techniques," NASA CR-111979, Dec. 1971.

[11] Chyung, D. H., "On the Problem of Controlling Certain Discrete Systems with Delays," *Presented at the IFAC Symposium on Systems Engineering Approach to Computer Control*, Paper 5.2, Kyoto, Japan, Aug. 1970.

[12] Mariani, L. and Nicoletti, B., "Optimal Discrete Systems with Pure Delays," *IEEE Transactions on Automatic Control*, Vol. AC-18, June 1973, pp. 311-313.

[13] Lasdon, L. S., Mitter, S., and Warren, A. D., "The Conjugate Gradient Method for Optimal Control Problems," *IEEE Transactions on Automatic Control*, Vol. AC-12, April 1967, pp. 132-138.

[14] Pagurek, B. and Woodside, C. M., "The Conjugate Gradient Method for Optimal Control Problems with Bounded Control Variables," *Automatica*, Vol. 4, 1968, pp. 337-349.